A non-linear model of cerebral diffusion: stability of finite differences method and resolution using the Adomian method

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Abstract *This paper describes a non-linear reaction-diffusion equation, which models how a substance spreads in the surface of the cortex so as to avoid a massive destruction of neurones when cerebral tissue is not oxygenated correctly. For the explicit finite differences method, the necessary stability condition is provided by a reaction-diffusion equation with non-linearity given by a decreasing function. The solution to the non-linear reaction-diffusion equation of the model can be obtained via one of the two methods: the finite differences (explicit schema) method and the Adomian method.*

1. Introduction

We present a biological model of the diffusion of a substance that prevents the death of the cerebral nerve cells when the cerebral tissue is no longer oxygenated correctly.

The regulation of nitric oxide (NO) in the human organism is a very topical subject at present. Nitric oxide is a biological regulator having a paradoxical nature. Given that it regulates blood flow and blood pressure, its presence is beneficial; in excess, however, it causes cellular death, by deregulating cellular respiration and producing inflammatory processes in the vascular wall, neuronal degeneration, etc. A deficiency in nitric oxide is a cardiovascular risk factor. Deserving researchers in respect to recent research on this subject are L. Ignarro, F. Murada and R. Furchgott, who received the Nobel Prize for Medicine in 1998 for their work on NO deficiency in the blood.

One method of cellular death is the formation of toxic substances in the cerebral tissue. The current hypothesis is that peroxynitrite is formed from nitric oxide/nitrites and from radical superoxide, both of which are the cells' response to anoxia (Beckman, 1995). In order to avoid this reaction in the cells, biologists have tried to restrict the formation of peroxynitrites.

The formation of nitrites in brain cells is an enzymatic phenomenon and the simplest method for preventing their formation is by using a specific enzyme

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inhibitor called NO-synthase. The inhibitor that is widely used is nitro-arginine (Iadecola *et al.*, 1994).

In order to penetrate the nitro-arginine to the brain of the rat (our animal model), we deposited a nitro-arginine solution directly onto the brain as the only way to ensure a constant concentration of nitro-arginine in the brain tissue (Greenberg *et al.*, 1997; Michel *et al.*, 1993; Tabrizi-Fard and Funy, 1996). Once the inhibitor was deposited on the rat's brain, it spread throughout the cerebral tissue and penetration to the cells relied on a transport mechanism of the Michaelis-Menten type (Bradbury, 1979).

2. Mathematical modelling

If we denote the concentration of extra-cellular inhibitor as $I(x,t)$, the equation that models this chemical phenomenon is a non-linear diffusion-reaction equation. This equation, in which the coefficient of diffusion has been calculated experimentally as $D = 0.0000038\ \mathrm{cm}^2/\mathrm{s}$, is as follows:

$$\frac{\partial I}{\partial t} = D\frac{\partial^2 I}{\partial x^2} - \frac{VI}{K+I} \quad t \geq 0,\ \ 0 \leq x \leq L,$$

where $K = 0.00001074\,\mathrm{mM/cm}^3$ and $V = 0.9865368 \times 10^{-7}\,\mathrm{mM/cm}^3/\mathrm{s/g}$. The initial and contour conditions are:

$$I(x,0) = C_0 \exp(-1,100x),$$

$$I(0,t) = C_0,\ \ C_0 = \text{constant} = 0.0091\,\mathrm{mM/cm}^3.$$

In this equation, the reaction term corresponds to the penetration of the inhibitor concentrate $I(x,t)$ within the cell, a penetration that takes place at Michaelis-Menten speed.

This equation has an unique solution (Rozier, 1984).

3. Solution using the finite differences method

The finite differences method is a classical approach to the solution of linear partial derivative equations. It is an approximate method, given that the partial derivatives in a point are approximated by a difference quotient over a small interval (Dautray and Lions, 1990; Euvrard, 1994; Golub and Ortega, 1993; Marcellán *et al.*, 1990). Depending on the expression of this difference quotient, there are various versions of the model, among them are the explicit, implicit and the Crank-Nicolson methods.

For the non-linear case, which is the equation for our model, there is no general proven theory. The procedure to follow depends on the type of non-linearity. There are occasions when a linearisation of the problem is recommended (Smith, 1978). But there are also cases when a generalisation of the methods indicated previously is possible. We demonstrate that under

certain hypotheses respecting f, the stability condition $r \leq 1/2$ for the linear case holds for the non-linear equation:

$$\frac{\partial u}{\partial t} = D\frac{\partial^2 u}{\partial x^2} + f(u).$$

If $f(u)$ is, in fact, decreasing, which happens when the non-linearity is of the Michaelis-Menten type, the explicit schema of the finite differences method is stable under the condition $r \leq 1/2$.

3.1 Necessary stability condition for the explicit finite differences method in the case of a reaction-diffusion equation with decreasing non-linearity

Theorem 3.1. Given the equation

$$\begin{aligned} &\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} + f(u(x,t)) \quad 0 \leq x \leq 1, \ \ t \geq 0, \\ &u(x,0) = u_0(x) \quad 0 \leq x \leq 1, \\ &u(0,t) = \alpha \qquad u(1,t) = \beta \quad \forall t \geq 0, \quad \alpha, \beta \text{ constants}, \end{aligned} \tag{1}$$

where $D > 0$ is the diffusion coefficient and $f(u)$ is a class C^1 non-linear function of u over an interval V of R such that $u(\mathrm{x}, \mathrm{t}) \in V$, $\forall (x,t) \in [0,1] \times [0, +\infty)]$ and f is decreasing; if $r \leq 1/2, r = D\Delta t/\Delta x^2$ and $f'(u) \geq 2r - 1/\Delta t$ $\forall u(x,t) \in V$, then the explicit finite differences schema is stable.

Proof. A whole number $N > 0$ is selected. Taking $\Delta x = 1/N$, $\Delta t > 0$ a grid is drawn in the interval $[0,1] \times [0, +\infty)$ of step size $\Delta x, \Delta t$. A generic point in the grid has the coordinates $(x_i, t_j) = (i\Delta x, j\Delta t)$. The approximation to the solution u of the equation in equation (1) for the point (x_i, t_j) is denoted by $u_{ij} = u(i\Delta x, j\Delta t)$ $i = 0, \ldots, N, j \geq 0$.

Extending the explicit schema of the finite differences method to the non-linear equation in equation (1), and denoting $k = \Delta t$, $h = \Delta x$, we have:

$$\begin{aligned} &\frac{u_{ij+1} - u_{ij}}{k} = D\frac{u_{i-1j} - 2u_{ij} + u_{i+1j}}{h^2} + f(u_{ij}) \quad i = 1, \ldots, N-1, \quad j > 0, \\ &u_{i0} = u_0(ih), \quad i = 1, \ldots, N-1, \\ &u_{0j} = \alpha, \quad u_{Nj} = \beta \quad \forall j \geq 0. \end{aligned} \tag{2}$$

Summarising we have:

$$u_{ij+1} = ru_{i-1j} + (1-2r)u_{ij} + ru_{i+1j} + kf(u_{ij}), \quad r = \frac{Dk}{h^2}.$$

The schema in equation (2) is expressed in matrix form as follows:

$$\begin{bmatrix} u_{1j+1} \\ u_{2j+1} \\ u_{3j+1} \\ \vdots \\ u_{N-1j+1} \end{bmatrix} = \begin{bmatrix} (1-2r) & r & & & & \\ r & (1-2r) & r & & & \\ & r & (1-2r) & r & & \\ & & \ddots & \ddots & \ddots & \\ & & & r & (1-2r) \end{bmatrix} \begin{bmatrix} u_{1j} \\ u_{2j} \\ u_{3j} \\ \vdots \\ u_{N-1j} \end{bmatrix}$$

$$+ k \begin{bmatrix} f(u_{1j}) \\ f(u_{2j}) \\ f(u_{3j}) \\ \vdots \\ f(u_{N-1j}) \end{bmatrix} + r \begin{bmatrix} \alpha \\ 0 \\ \vdots \\ 0 \\ \beta \end{bmatrix}.$$

In other words,

$$\mathbf{u}_{j+1} = \mathbf{A}\mathbf{u}_j + k\mathbf{F}(\mathbf{u}_j) + r\mathbf{B}, \tag{3}$$

where the following notation has been used:

$$\mathbf{u}_j = \begin{bmatrix} u_{1j} \\ u_{2j} \\ u_{3j} \\ \vdots \\ u_{N-1j} \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} (1-2r) & r & & & \\ r & (1-2r) & r & & \\ & r & (1-2r) & r & \\ & & \ddots & \ddots & \ddots \\ & & & r & (1-2r) \end{bmatrix},$$

$$\mathbf{F}(\mathbf{u}_j) = \begin{bmatrix} f(u_{1j}) \\ f(u_{2j}) \\ f(u_{3j}) \\ \vdots \\ f(u_{N-1j}) \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \alpha \\ 0 \\ \vdots \\ 0 \\ \beta \end{bmatrix}.$$

Represented by $\mathbf{u}_{j+1}$ is the solution vector obtained by the explicit schema for finite differences in the stage $j+1$, starting off from the initial vector $\mathbf{u}_0$. Let us suppose that a small perturbation is introduced into $t = 0$, such that $\mathbf{u}_0^* \leq \mathbf{u}_0$,

on the understanding that the inequality is verified on a component-by-component basis.

Before we look at the stability, we prove that if $\mathbf{u}_j^* \leq \mathbf{u}_j$ in the stage j, then $\mathbf{u}_{j+1}^* \leq \mathbf{u}_{j+1}$. We will start the calculations using the schema in equation (3), with the vector of initial values $\mathbf{u}_0^*$ such that $\mathbf{u}_0^* \leq \mathbf{u}_0$. We thus have

$$\mathbf{u}_{j+1}^* = \mathbf{A}\mathbf{u}_j^* + k\mathbf{F}(\mathbf{u}_j^*) + r\mathbf{B}.$$

We define the error vector $\mathbf{e} = \mathbf{u} - \mathbf{u}^*$; thus,

$$\mathbf{e}_{j+1} = \mathbf{u}_{j+1} - \mathbf{u}_{j+1}^* = \mathbf{A}(\mathbf{u}_j - \mathbf{u}_j^*) + k(\mathbf{F}(\mathbf{u}_j) - \mathbf{F}(\mathbf{u}_j^*)). \tag{4}$$

We want to prove that if $\mathbf{e}_j \geq 0$ then $\mathbf{e}_{j+1} \geq 0$, where 0 denotes the null vector with dimensions $N - 1$.

Given that f is class C^1, then $f(u_{ij}) - f(u_{ij}^*) = f'(w_{ij})(u_{ij} - u_{ij}^*)$, with w_{ij} intermediate between u_{ij} and u_{ij}^*. Thus, if we denote the diagonal matrix, $\mathbf{F}'(\mathbf{w}_j) = \text{diag}(f'(w_{1j}), \ldots, f'(w_{N-1j}))$, the expression in equation (4) is converted into:

$$\mathbf{e}_{j+1} = \mathbf{A}(\mathbf{u}_j - \mathbf{u}_j^*) + k\mathbf{F}'(\mathbf{w}_j)(\mathbf{u}_j - \mathbf{u}_j^*).$$

In other words,

$$\mathbf{e}_{j+1} = (\mathbf{A} + k\mathbf{F}'(\mathbf{w}_j))\mathbf{e}_j. \tag{5}$$

We will denote as $\mathbf{B}_j$ the matrix $\mathbf{A} + k\mathbf{F}'(\mathbf{w}_j)$, and thus equation (5) becomes:

$$\mathbf{e}_{j+1} = \mathbf{B}_j\mathbf{e}_j. \tag{6}$$

Since under the hypothesis

$$f'(u) \geq \frac{2r-1}{k} \quad \forall u(x,t) \in V,$$

the proof that $\mathbf{B}_j \geq 0_{N-1}$, where 0_{N-1} denotes the null matrix of dimensions $N - 1$, is immediate. Thus,

$$\text{if } \mathbf{e}_j \geq 0 \text{ then } \mathbf{e}_{j+1} \geq 0. \tag{7}$$

In order to prove stability, we limit the expression in equation (6). Since f is decreasing, $f'(u) \leq 0\ \forall u \in V$, which implies that $\mathbf{F}'(\mathbf{w}_j) \leq 0_{N-1}$. And so $\mathbf{B}_j \leq \mathbf{A}$ and since under the hypothesis $r \leq 1/2$, then $\mathbf{A} \geq 0_{N-1}$. Applying equation (7) we have:

$$\mathbf{e}_{j+1} \leq \mathbf{A}\mathbf{e}_j.$$

Repeating the reasoning recursively,

$$\mathbf{e}_{j+1} \leq \mathbf{A}\mathbf{e}_j \leq \mathbf{A}^2\mathbf{e}_{j-1} \leq \ldots \leq \mathbf{A}^{j+1}\mathbf{e}_0. \tag{8}$$

The explicit schema of finite differences will be stable if $\mathbf{e}_j$ tends to the vector 0 and if j grows indefinitely.

If we denote $\rho(\mathbf{A})$ as the spectral radius of the matrix $\mathbf{A}$, we know that if

$$\rho(\mathbf{A}) = \max_{i=1,\ldots,N-1} |\lambda_i| < 1,$$

then

$$\lim_{j \to \infty} A^j = 0_{N-1},$$

where the values corresponding to the matrix A are represented by λ_i. Therefore, if $\rho(\mathbf{A}) < 1$, from equation (8) we can conclude that the schema will be stable.

Since the values for $\mathbf{A}$ are:

$$\lambda_i = 1 - 2r + 2r\cos\frac{i\pi}{N}, \quad i = 1, \ldots N-1,$$

if $0 < r \leq 1/2$, then it is verified that

$$-2r < 2r\cos\frac{i\pi}{N} < 2r,$$

and thus,

$$-1 < 1 - 2r - 2r < 1 - 2r + 2r\cos\frac{i\pi}{N} < 1 - 2r + 2r = 1.$$

In other words, $|\lambda_i| < 1 \ \ \forall i = 1, \ldots, N-1$. And so, if $r \leq 1/2$, the schema will be stable. □

Corollary 3.1. Given the equation:

$$\begin{aligned} &\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{Vu(x,t)}{K + u(x,t)} \quad 0 \leq x \leq 1, \ \ t \geq 0 \\ &u(x,0) = u_0(x) \quad 0 \leq x \leq 1 \\ &u(0,t) = \alpha, \qquad u(1,t) = \beta \quad \forall t \geq 0, \quad \alpha, \beta \text{ constants}, \end{aligned} \tag{9}$$

where D > 0 is the diffusion coefficient and V and K are positive constants; if $r \leq 1/2$, $r = D\Delta t/\Delta x^2$, then the explicit finite differences schema is stable.

Proof. This is an immediate consequence of Theorem 3.1. Equation (9) is a special case of equation (1) given that:

$$f(u) = -\frac{Vu}{K+u}$$

is decreasing. □

3.2 Numerical solution using the explicit finite differences method

This is a question of solving, using the explicit schema of the finite differences method, the following equation:

$$\frac{\partial I}{\partial t} = 0.0000038\,\frac{\partial^2 I}{\partial x^2} - \frac{VI}{K+I} \quad 0 \le x \le 0.15, \quad t \ge 0$$

with the initial and contour conditions:

$$I(x,0) = 0.0091\exp(-1100x) \quad I(0,t) = 0.0091, \quad I(0.15,\, t) = 0.$$

The equation discretized by the method is:

$$u_{ij+1} = ru_{i-1j} + (1-2r)u_{ij} + ru_{i+1j} - k\frac{Vu_{ij}}{K+u_{ij}}, \quad r = \frac{0.0000038k}{h^2},$$

with the initial and contour conditions:

$$u_{i0} = 0.0091\exp(-1100ih), \quad i = 1,\ldots,N-1,$$

$$u_{0j} = 0.0091, \quad u_{Nj} = 0 \quad \forall j \ge 0.$$

For V and K we will take the values $V = 0.9865368 \times 10^{-7}$ and $K = 0.00001074$ expressed in Section 2.

The condition $r \le 1/2$ ensures the stability of the system.

3.2.1 Numerical results. Taking $\Delta t = 0.5$, $\Delta x = 0.00625$ and $D = 3.8 \times 10^{-6}$, we obtain a value for $r = D\Delta t/\Delta x^2 = 0.048640$, and thus the solution is stable. The numerical solution is obtained via MAPLE V and shown graphically in Figure 1.

The graph represents the diffusion phenomenon modelled using level curves drawn for several instants of time t: that curves have been calculated every 0.5 s for 20 min and that all of these are shown. The Y axis represents the concentrations of the inhibitor I, and the X axis represents cortex depth x.

4. Solution using the Adomian method

4.1 Classical presentation of the Adomian method

This method was proposed by the North American physicist, G. Adomian (1923-1996). It is based on the search for a solution in the form of a series and on decomposing the non-linear operator into a series in which the terms are calculated recurrently using Adomian polynomials. Under certain conditions of

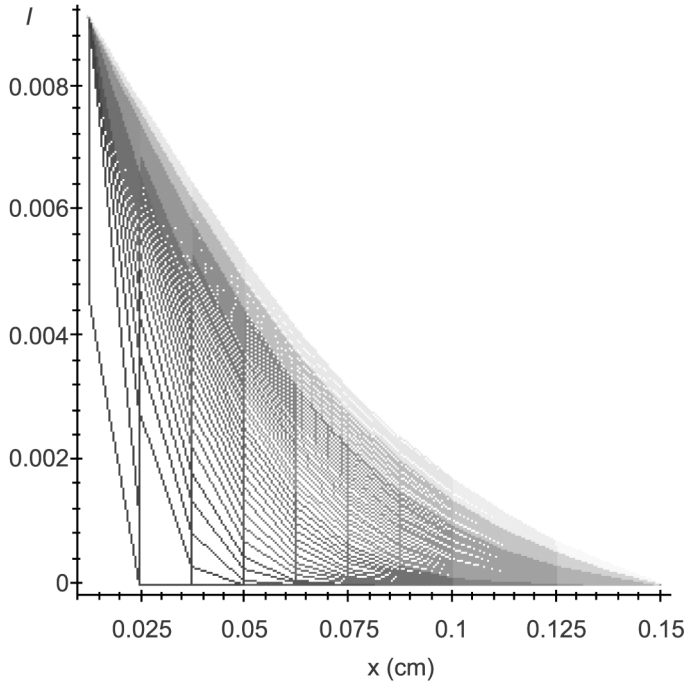

Figure 1.

convergence, the sum of the series will give an exact solution, but in practice the series will be truncated and yet give a good approximation. The truncation error can generally be calculated.

Let us consider the non-linear functional equation (Adomian, 1983, 1986, 1994; Bellman and Adomian, 1985)

$$u - N(u(t)) = f(t), \tag{10}$$

where N represents a known non-linear operator, f is given for a Banach space H (N operator of $H \times H$).

The problem is to determine the solution $u \in \mathrm{H}$ of equation (10). The Adomian method consists of searching for a solution u, if it exists, in the form of a series

$$u = \sum_{n=0}^{\infty} u_n, \tag{11}$$

and decomposing the non-linear term $N(u)$ in the form

$$N(u) = \sum_{i=0}^{\infty} A_i. \tag{12}$$

The A_i are polynomials – called Adomian polynomials – that depend exclusively on $u_0, u_1, \ldots, u_n$ and that are obtained from the equations

$$v = \sum_{i=0}^{\infty} \lambda^i u_i, \quad N\left(\sum_{i=0}^{\infty} \lambda^i u_i\right) = \sum_{i=0}^{\infty} \lambda^i A_i\,, \tag{13}$$

where λ is a parameter introduced for convenience sake. The A_n can be obtained formally from equation (4) (Bellman and Adomian, 1985) using the expression

$$n! A_n = \frac{\mathrm{d}^n}{\mathrm{d}\lambda^n}\left[N\left(\sum_{i=0}^{n} \lambda^i u_i\right)\right]_{\lambda=0} \tag{14}$$

Replacing equations (11) and (12) in equation (10), we obtain

$$\sum_{n=0}^{\infty} u_n - \sum_{n=0}^{\infty} A_n = f \tag{15}$$

The terms of the series $\sum_{n=0}^{\infty} u_n$ are obtained by identification in equation (15):

$$\begin{cases} u_0 = f \\ u_1 = A_0 \\ \vdots \\ u_{n+1} = A_n \end{cases} \tag{16}$$

The series that is the solution of equation (10) is thus determined. Equation (14) which defines the A_j shows that the A_j depend only on $u_0, u_1, \ldots, u_j$ and not on $u_{j+1}, u_{j+2}, \ldots$. For scalar N functions, the A_n can be found (Abbaoui, 1995) by:

$$\begin{cases} A_0(u_0) = N(u_0) \\ A_n(u_0, \ldots, u_n) = \sum_{\alpha_1 + \ldots + \alpha_n = n} N^{(\alpha_1)}(u_0) \dfrac{u_1^{(\alpha_1 - \alpha_2)}}{(\alpha_1 - \alpha_2)!} \cdots \dfrac{u_{n-1}^{(\alpha_{n-1} - \alpha_n)}}{(\alpha_{n-1} - \alpha_n)!} \cdot \dfrac{u_n^{\alpha_n}}{\alpha_n!} \quad n \neq 0 \end{cases}$$

where the succession $(\alpha_i)_{i=1,\ldots,n}$ is decreasing.

More information on the Adomian method can be found in Cherruault and Adomian (1993) and Cherruault *et al.* (1995). Applications to systems of non-linear differential equations are discussed in Grimalt and Pujol (1999) and Guellal *et al.* (1997), where there is an application to kinetic chemistry. For applications to partial differential equations see Guellal *et al.* (2000).

4.2 Resolution of the non-linear diffusion-reaction equation using the Adomian method

Let us consider the equation

$$\frac{\partial I}{\partial t} = 0.0000038\frac{\partial^2 I}{\partial x^2} - \frac{VI}{K+I} \quad t \geq 0, \;\; 0 \leq x \leq L, \tag{17}$$

with the initial and contour conditions

$$I(x,0) = C_0 \exp(-1100x),$$

$$I(0,t) = C_0, \;\; C_0 = \text{constant} = 0.0091\,\text{mM/cm}^3.$$

Equation (17) is written in canonical form as:

$$L_t I = 0.0000038 L_{xx} I - N(I), \tag{18}$$

where $L_t I = \partial I/\partial t$, $L_{xx} I = \partial^2 I/\partial x^2$ and $N(I) = VI/K + I$.

We invert the lesser order operator L_t. In this case, the Adomian schema is written as:

$$\forall n \geq 1 \quad I_n = 0.0000038 L_t^{-1} L_{xx} I_{n-1} - L_t^{-1} A_{n-1}.$$

where L_t^{-1} represents the integration in t and A_{n-1} are the Adomian polynomials corresponding to N non-linearity and depending on $I_0, I_1, \ldots, I_{n-1}$.

The Adomian schema, based on equations (15) and (16), is written as:

$$\sum_{n=0}^{\infty} I_n = I(x,0) + 0.0000038 L_t^{-1} L_{xx} \sum_{n=0}^{\infty} I_n - L_t^{-1} \sum_{n=0}^{\infty} A_n,$$

with

$$\begin{aligned} N(I) &= \sum_{n=0}^{\infty} A_n = \frac{VI}{K+I} \\ I_0 &= I(x,0) = 0.0091 \exp(-1100x) \\ I_{n+1} &= 0.0000038 L_t^{-1} L_{xx} I_n - L_t^{-1} A_n \end{aligned} \tag{19}$$

and the A_n are calculated by equation (14).

Therefore, applying equation (19),

$$I_0 = 0.0091 \mathrm{e}^{-1100x}$$

$$I_1 = 0.0418418 t \mathrm{e}^{-1100x} - 91\frac{tV}{10000K\mathrm{e}^{1100x} + 91}$$

$$I_2 = t^2(0.961942982 \times 10^{11}\mathrm{e}^{2200x}K^3 + 0.2626104341 \times 10^{10}\mathrm{e}^{1100x}K^2 + 0.2389754950 \times 10^8 K + 0.7248923349 \times 10^5\mathrm{e}^{-1100x} - 0.418418 \times 10^{11}\mathrm{e}^{2200x}VK^2 + 0.455 \times 10^{10}V^2K\mathrm{e}^{2200x})/(10000K\mathrm{e}^{1100x} + 91)^3$$

and thus the sum of the truncated series gives the following expression for I:

$$I = 0.0091\mathrm{e}^{-1100x} + 0.0418418t\mathrm{e}^{-1100x} - 91\frac{tV}{10000K\mathrm{e}^{1100x} + 91} + t^2(0.961942982 \times 10^{11}\mathrm{e}^{2200x}K^3 + 0.2626104341 \times 10^{10}\mathrm{e}^{1100x}K^2 + 0.2389754950 \times 10^8 K + 0.7248923349 \times 10^5\mathrm{e}^{-1100x} - 0.418418 \times 10^{11}\mathrm{e}^{2200x}VK^2 + 0.455 \times 10^{10}V^2K\mathrm{e}^{2200x})/(10000K\mathrm{e}^{1100x} + 91)^3 \quad (20)$$

The convergence of the series is the consequence of a theorem (Abbaoui *et al.*, 2001) in which the nth derivative of the non-linear term $N(I) = VI/K + I$ is required to be bounded.

5. Comparison of results

The first point of note is that both the solution obtained by the finite differences method is numerical with space and time discretization, whereas the solution provided by the Adomian method does not discretize space or time and gives an analytical solution in the form of a truncated series.

It has been demonstrated that both the methods produce results that are highly satisfactory, in comparison with the experimental results for concentrations of the inhibitor I obtained by microvoltametry in the laboratory. For example, for a depth of $x = 0.0125$ cm, the results in Table I were obtained.

Seconds	Experimental I (mM/cm^3)	Adomian I (mM/cm^3)	Dif finite I (mM/cm^3)
0	0	$0.9716106491 \times 10^{-8}$	0
60	0.000370	0.0003709583422	0.000369648
120	0.001475	0.001478453962	0.001479623
180	0.003351	0.003322496574	0.003295641
240	0.005910	0.005903086180	0.005900843

Table I. Comparison of experimental results and those obtained via the Adomian and finite differences methods

6. Conclusions

A diffusion model is described for a cerebral bio-chemistry problem, based on a non-linear partial derivative equation.

We have described the solution to the equation obtained by a classical resolution method in non-linear partial derivative equations – the finite differences method. We have proved that the said method may also be extended to non-linear reaction-diffusion partial derivative equations when non-linearity is a decreasing function. The solution obtained is numerical with space and time discretization.

The Adomian method is an effective tool for the resolution of certain kinds of non-linear partial derivative equations. The equation for the model is solved using the Adomian method, which does not discretize space or time and gives the analytical solution in the form of a truncated series.

It has been demonstrated that both methods produce results that are highly satisfactory, in comparison with the experimental results for concentrations of the inhibitor I obtained by microvoltametry in the laboratory.

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